One-sided alternative

two-sided	$H_0 : \mu = \mu_0$	$H_1 : \mu \neq \mu_0$
one-sided	$H_0 : \mu \geq \mu_0$	$H_1 : \mu < \mu_0$

Example 3

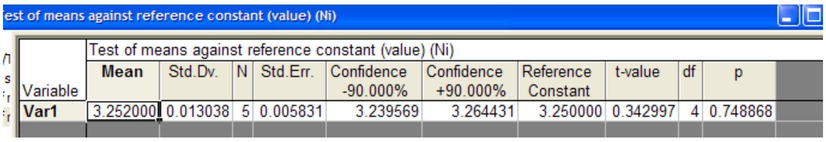
Ni content of a solution is measured: 3.25, 3.27, 3.24, 3.26, 3.24
The required minimum value is 3.25 g/cm^3
Is it reached?

$H_0 : \mu \geq \mu_0$ $H_1 : \mu < \mu_0$ (minimum value is not reached)

$$t_0 = \frac{\bar{x}-\mu_0}{s/\sqrt{n}} = \underbrace{\frac{\bar{x}-\mu}{s/\sqrt{n}}}_{t} + \underbrace{\frac{\mu-\mu_0}{s/\sqrt{n}}}_{<0 \text{ if } H_1 \left(\mu < \mu_0\right)}$$

lower tail, lower limit

$\mathrm{H}_0 : \mu \geq \mu_0$ $\mathrm{H}_1 : \mu < \mu_0$

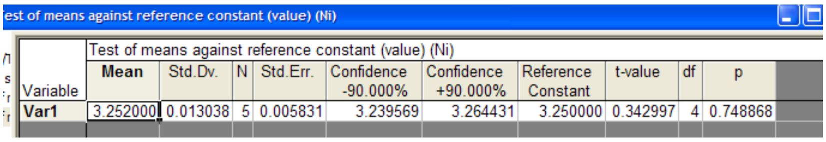


est of means against reference constant (value) (Ni)

Variable	Test of means against reference constant (value) (Ni) Mean	Std.Dv.	N	Std.Err.	Confidence -90.000%	Confidence +90.000%	Reference Constant	t-value	df	p
Var1	3.252000	0.013038	5	0.005831	3.239569	3.264431	3.250000	0.342997	4	0.748868

The hypothesised μ_0=3.25 value is above the lower confidence limit (p>0.05), accepted:
failed to reject ...
not proved that at most...

$\mathrm{H}_0 : \mu \leq \mu_0$ $\mathrm{H}_1 : \mu > \mu_0$



est of means against reference constant (value) (Ni)

Variable	Test of means against reference constant (value) (Ni) Mean	Std.Dv.	N	Std.Err.	Confidence -90.000%	Confidence +90.000%	Reference Constant	t-value	df	p
Var1	3.252000	0.013038	5	0.005831	3.239569	3.264431	3.250000	0.342997	4	0.748868

The hypothesised μ_0=3.25 value is below the upper confidence limit (p>0.05), accepted:
failed to reject ...
not proved that at most...

If the null hypothesis is accepted it does not „prove" that it is right.
If you want to prove something (e.g. the minimum Ni content is achieved) you have to put it in the null hypothesis.

Comparing two variances
(*F* test)

$H_0 : \sigma_1^2 = \sigma_2^2$

The test statistic: $F_0 = \frac{s_1^2}{s_2^2};(n_1 - 1, n_2 - 1)$

In case of one-sided alternative: $H_1 : \sigma_1^2 > \sigma_2^2$

The null hypothesis is rejected if $s_1^2 / s_2^2 > F_\alpha$

In case of two-sided alternative: $H_1 : \sigma_1^2 \neq \sigma_2^2$

The null hypothesis is rejected if

$$\frac{s_1^2}{s_2^2} < F_{1\text{-}a/\ 2} \quad \text{or} \quad \frac{s_1^2}{s_2^2} > F_{a/\ 2}$$

using $s_1^2 / s_2^2 \geq 1$ it is sufficient to check the upper limit

Two-sample *t* test

Two independent samples $n_1, n_2; \quad s_1^2, s_2^2; \quad \bar{x}_1, \bar{x}_2$

$$\mathrm{H}_0 : \ E(\bar{x}_1 - \bar{x}_2) = \mu_1 - \mu_2 = 0$$

Assuming the equality of variance for the two populations (to be checked through F-test):

$$\sigma_1^2 = \sigma_2^2$$

$$\mathrm{H}_0 : \ E(\bar{x}_1 - \bar{x}_2) = \mu_1 - \mu_2 = 0$$

$$t = \frac{\bar{x}_1 - \bar{x}_2 - E(\bar{x}_1 - \bar{x}_2)}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$\nu = n_1 + n_2 - 2$$

$$s^2 = \frac{1}{n_1 + n_2 \text{-} 2}\left[s_1{}^2(n_1 - 1) + s_2{}^2(n_2 - 1)\right]$$

The test statistic:

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad , \qquad \nu = (n_1 - 1) + (n_2 - 1)$$

The assumption $\sigma_1^2 = \sigma_2^2$ is checked through *F* test

---

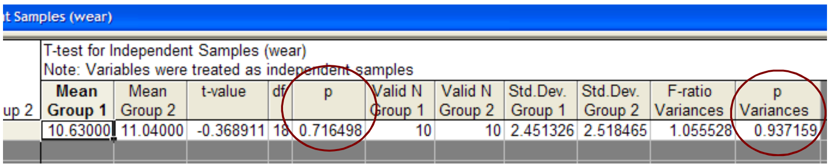
Example 4

(Box-Hunter-Hunter: Statistics for Experimenters, J. Wiley, 1978, p. 97)

The wear of two kinds of raw material is compared as shoe soles on the foot of 10-10 boys (shoes1.xls).

Is the difference of means and variances significant at 0.05 level?

	n	mean	sample variance
A	10	10.63	6.009
B	10	11.04	6.343



t Samples (wear)

T-test for Independent Samples (wear)
Note: Variables were treated as independent samples

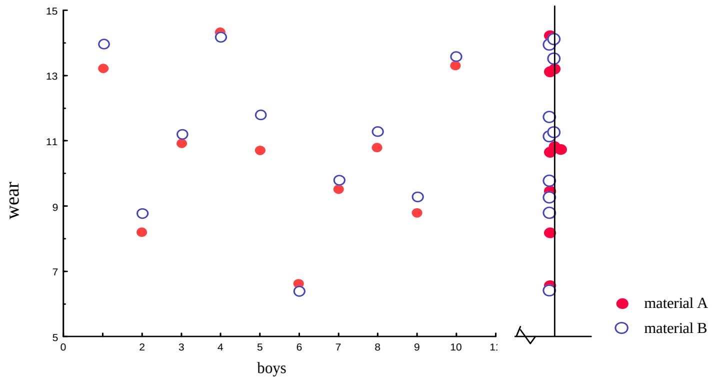
up 2	**Mean Group 1**	Mean Group 2	t-value	df	p	Valid N Group 1	Valid N Group 2	Std.Dev. Group 1	Std.Dev. Group 2	F-ratio Variances	p Variances
	10.63000	11.04000	-0.368911	18	0.716498	10	10	2.451326	2.518465	1.055528	0.937159

Example 15

(Box-Hunter-Hunter: Statistics for Experimenters, J. Wiley, 1978, p. 97)

TABLE 4.3. Data on the amount of wear measured with two different materials A and B, boy's shoes example*

boy	material A	material B	B – A difference *d*
1	13.2(L)	14.0(R)	0.8
2	8.2(L)	8.8(R)	0.6
3	10.9(R)	11.2(L)	0.3
4	14.3(L)	14.2(R)	-0.1
5	10.7(R)	11.8(L)	1.1
6	6.6(L)	6.4(R)	-0.2
7	9.5(L)	9.8(R)	0.3
8	10.8(L)	11.3(R)	0.5
9	8.8(R)	9.3(L)	0.5
10	13.3(L)	13.6(R)	0.3
		average difference	0.41



Paired *t* test

$\mathrm{H}_0 : E(x_i) = E(y_i) = 0$

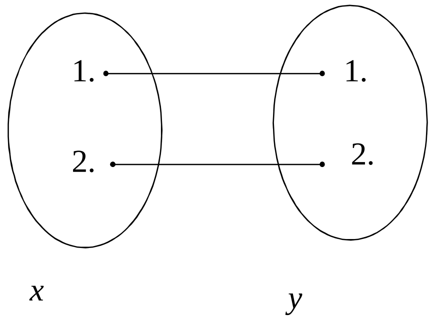
$\mathrm{H}_0 :\ E(d_i) = E(x_i) - E(y_i)$

$d_i = x_i - y_i$ one-sample *t* test for the differences

$$\bar{d} = \frac{\sum_i d_i}{n} \qquad s_d^2 = \frac{\sum_i \left(d_i \text{-} \bar{d}\right)^2}{n\text{-}1} \qquad t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

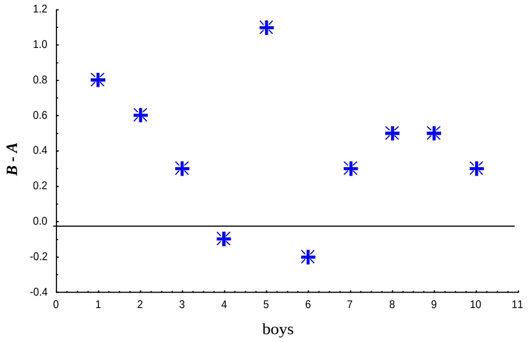
Paired *t* test

$H_0 : E(d) = 0$ $d_i = x_i - y_i$



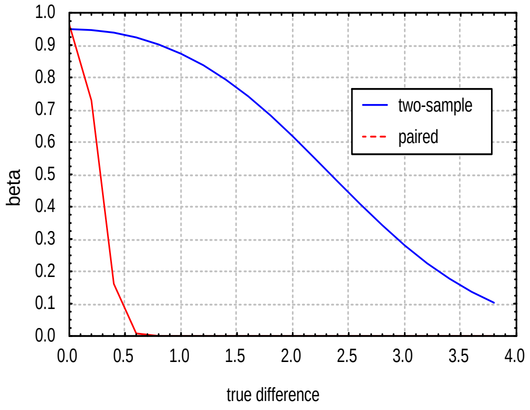
dependent samples

$$s_d^2 = 0.149 \qquad s_d = \sqrt{0.149} = 0.386 \qquad \frac{s_d}{\sqrt{n}} = \frac{0.386}{\sqrt{10}} = 0.122$$

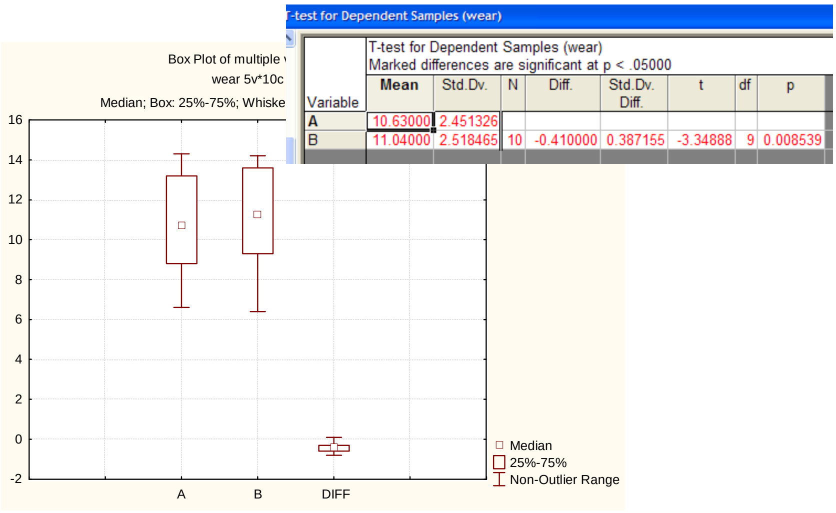


$$t_0 = \frac{0.41}{0.122} = 3.4$$

OC curve for the boys shoes example



Statistics > Basic Statistics/Tables
t-test, dependent samples



T-test for Dependent Samples (wear)
Marked differences are significant at p < .05000

Variable	Mean	Std.Dv.	N	Diff.	Std.Dv. Diff.	t	df	p
A	10.63000	2.451326						
B	11.04000	2.518465	10	-0.410000	0.387155	-3.34888	9	0.008539

Testing goodness of fit

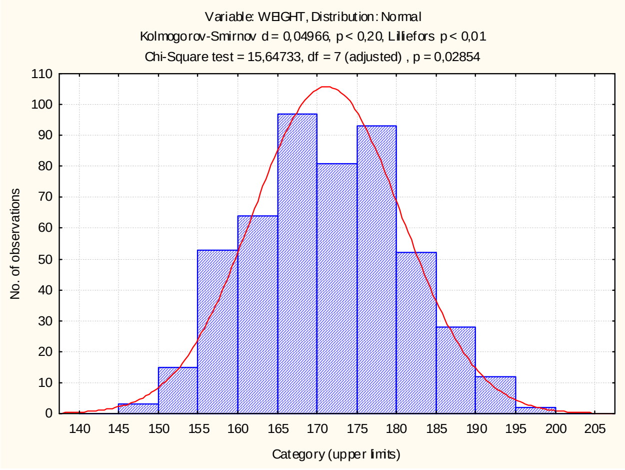
It is to be judged if the data may come from a certain distribution

Normality test

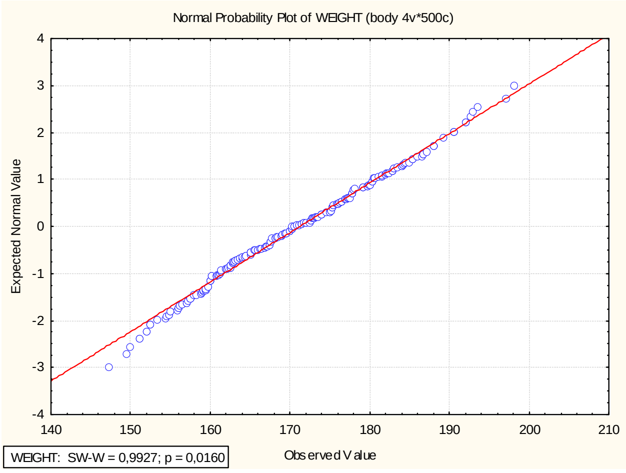
graphical tests

statistical tests

Open: *body.sta*
Statistics > Distribution fitting > continuous Distributions > Normal



Graphs > 2D Graphs > Normal Probability Plots...



Statistical tests for goodness of fit

Large samples:

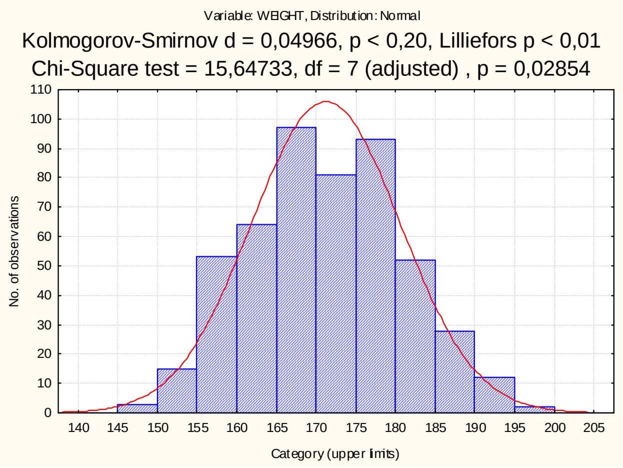
Kolmogorov-Smirnov test
The data are grouped into classes, at least 5 classes are required.

χ^2 -test
The data are grouped into classes, at least 5 occurrences are required in a class.

Smaller samples:

Shapiro-Wilk test

Statistics > Distribution fitting > continuous Distributions > Normal
Options tab
Kolmogorov-Smirnov test: yes(continuous)

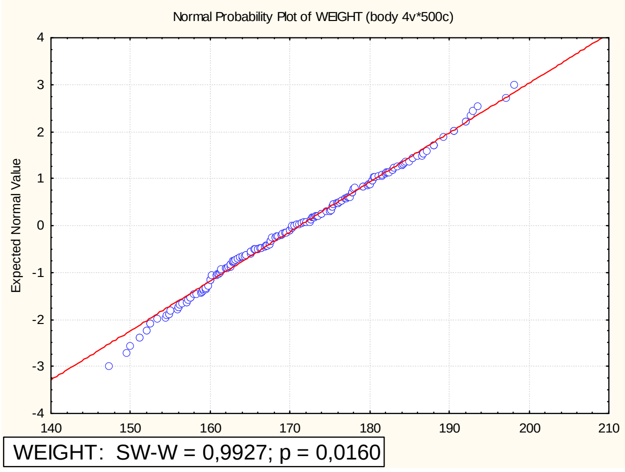


Variable: WEIGHT, Distribution: Normal

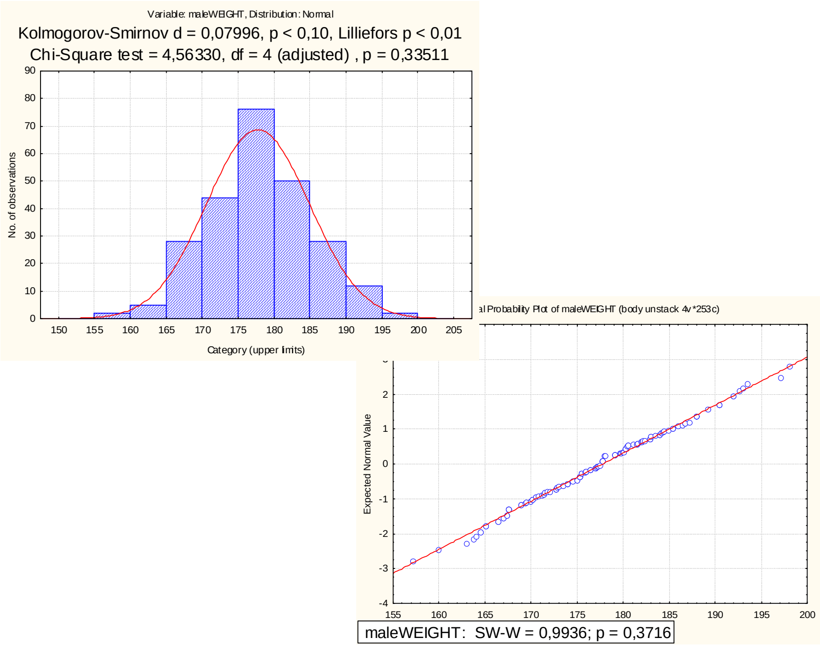
Kolmogorov-Smirnov d = 0,04966, p < 0,20, Lilliefors p < 0,01

Chi-Square test = 15,64733, df = 7 (adjusted) , p = 0,02854

Graphs > 2D Graphs > Normal Prbability Plots…
Shapiro-Wilk test



WEIGHT: SW-W = 0,9927; p = 0,0160



Kolmogorov-Smirnov d = 0,07996, p < 0,10, Lilliefors p < 0,01
Chi-Square test = 4,56330, df = 4 (adjusted) , p = 0,33511

maleWEIGHT: SW-W = 0,9936; p = 0,3716