$$\left(y_{ij}-Y_i\right)=\left(y_{ij}-\hat{Y}_i\right)+\left(\hat{Y}_i-Y_i\right)=\left(y_{ij}-\hat{\mu}-\hat{\alpha}_i\right)+\left(\hat{\mu}-\mu\right)+\left(a_i-\hat{\alpha}_i\right)$$

$$\sum_i\sum_j\left(y_{ij}-Y_i\right)^2=\sum_i\sum_j\left(y_{ij}-\hat{\mu}-\hat{\alpha}_i\right)^2+\left(\hat{\mu}-\mu\right)^2\sum_i p_i+\sum_i p_i\left(\hat{\alpha}_i-\alpha_i\right)^2=$$

$$=\sum_i\sum_j\left(y_{ij}-y_{i\cdot}\right)^2+\left(y_{\cdot\cdot}-\mu\right)^2\sum_i p_i+\sum_i p_i\left(y_{i\cdot}-y_{\cdot\cdot}-\alpha_i\right)^2$$

Fisher-Cochran-tétel $\sum_i p_i=\sum_i\left(p_i-1\right)+1+\left(r-1\right)$

mind $\chi^2\sigma_e^2$

$$\sum_i p_i\left(y_{i\cdot}-y_{\cdot\cdot}-\alpha_i\right)^2=\chi^2\sigma_e^2$$

$H_0:\alpha_i=0$ $\longrightarrow$ $S_A=\sum_i p_i\left(y_{i\cdot}-y_{\cdot\cdot}\right)^2=\chi^2\sigma_e^2$

$$S_R=\sum_i\sum_j\left(y_{ij}-y_{i\cdot}\right)^2=\chi^2\sigma_e^2$$

$$s_A^2=\frac{\sum_{i=1}^{r} p_i\left(y_{i\cdot}-y_{\cdot\cdot}\right)^2}{r-1}=\frac{\chi^2\sigma_e^2}{\nu_A}\qquad s_R^2=\frac{\sum_i\sum_j\left(y_{ij}-y_{i\cdot}\right)^2}{\sum_i p_i-r}=\frac{\chi^2\sigma_e^2}{\nu_R}$$

$$F_0=\frac{s_A^2}{s_R^2}$$

Két faktor szerinti ANOVA

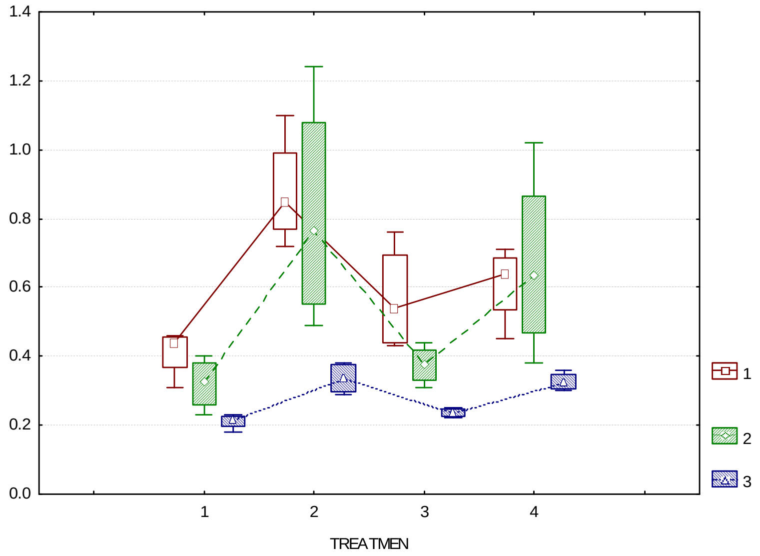
Az *A* faktor minden szintjét kombináljuk a *B* faktor minden szintjével, minden „cellában” azonos számú ismétlés (kiegyensúlyozott terv).
A terv szerkezete miatt a faktorok hatását egymásétól függetlenül vizsgálhatjuk.

2. példa

(Box-Hunter-Hunter: Statistics for Experimenters, J. Wiley, 1978, p. 165)
poison.sta

		treatment			
		A	B	C	D
poison	I	0.310	0.820	0.430	0.450
		0.450	1.100	0.450	0.710
		0.460	0.880	0.630	0.660
		0.430	0.720	0.760	0.620
	II	0.360	0.920	0.440	0.560
		0.290	0.610	0.350	1.020
		0.400	0.490	0.310	0.710
		0.230	1.240	0.400	0.380
	III	0.220	0.300	0.230	0.300
		0.210	0.370	0.250	0.360
		0.180	0.380	0.240	0.310
		0.230	0.290	0.220	0.330

ANOVA



Modell

i=1,…,r; j=1,…,q, k=1,…,p

$$y_{ijk} = \mu_{ij} + \varepsilon_{ijk}$$

$$N = rqp = 3 \cdot 4 \cdot 4 = 48$$

ismétlés (k)

méreg (i) kezelés (j)

$$y_{ijk} = \mu_{ij} + \varepsilon_{ijk}$$ átlag-modell

$$\mathrm{H}_0 : \mu_{11} = \mu_{12} = \cdots = \mu_{34}$$ (mind azonos)

ANOVA

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk}$$

az *i*-edik méreg hatása — a *j*-edik kezelés hatása — kölcsönhatás

$$y_{ijk} = \mu + \alpha_i + \beta_j + \alpha\beta_{ij} + \varepsilon_{ijk}$$ hatás-modell

$$H_0^A : \alpha_i = 0,\ i = 1,\dots, r$$

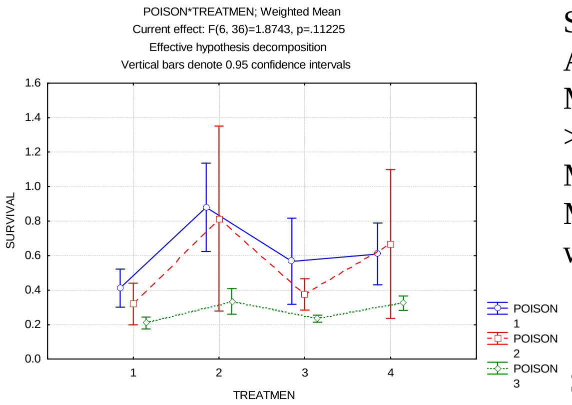
$$H_0^B : \quad \beta_j = 0,\ j = 1,\dots, q$$

$$H_0^{AB} : \quad \alpha\beta_{ij} = 0,\ i = 1,\dots, r;\ j = 1,\dots, q$$

ANOVA-táblázat

Az eltérés forrása	eltérés- -négyzetösszeg	szabadsági fok	szórásnégyzet	F
A hatása (sorok közötti)	$S_A = qp\sum_i \left(y_{i\cdot\cdot} - y_{\cdots}\right)^2$	r-1	$s_A^2 = \dfrac{S_A}{r-1}$	s_A^2 / s_R^2
B hatása (oszlopok közötti)	$S_B = rp\sum_j \left(y_{\cdot j\cdot} - y_{\cdots}\right)^2$	q-1	$s_B^2 = \dfrac{S_B}{q-1}$	s_B^2 / s_R^2
AB kölcsönhatás	$S_{AB} =$ $= p\sum_i\sum_j \left(y_{ij\cdot} - y_{i\cdot\cdot} - y_{\cdot j\cdot} + y_{\cdots}\right)^2$	$(r$-1)·(q-1)	$s_{AB}^2 = \dfrac{S_{AB}}{(r-1)(q-1)}$	s_{AB}^2 / s_R^2
Maradék (csoportokon belüli)	$S_R = \sum_i\sum_j\sum_k \left(y_{ijk} - y_{ij\cdot}\right)^2$	$rq(p$-1)	$s_R^2 = \dfrac{S_R}{rq(p-1)}$	
Teljes	$S_0 = \sum_i\sum_j\sum_k \left(y_{ijk} - y_{\cdots}\right)^2$	rqp-1		

3·4·(4-1)=36



Statistics>
Advanced Linear/Nonlinear Models>
>General Linear Models>Factorial ANOVA>
Means fülön: Observed, weighted, Plot

Summary fülön: All effects

Univariate Tests of Significance for SURVIVAL (Poison)
Sigma-restricted parameterization
Effective hypothesis decomposition

Effect	SS	Degr. of Freedom	MS	F	p
Intercept	11.03042	1	11.03042	495.9194	0.000000
POISON	1.03301	2	0.51651	23.2217	0.000000
TREATMEN	0.92121	3	0.30707	13.8056	0.000004
POISON*TREATMEN	0.25014	6	0.04169	1.8743	0.112251
Error	0.80073	36	0.02224		

Homoszkedaszticitás $\sigma_e^2 = \text{konst} \quad ?$

More results>Assumptions fülön: Homogeneity of variances

Tests of Homogeneity of Variances (Poison)
Effect: POISON*TREATMEN

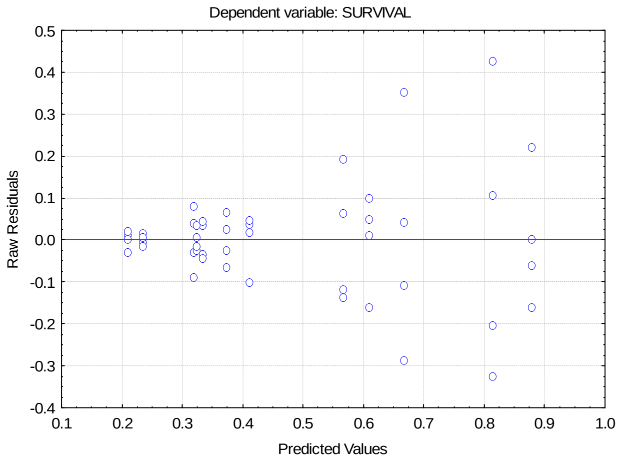
	Hartley F-max	Cochran C	Bartlett Chi-Sqr.	df	p
SURVIVAL	678.6000	0.423741	45.13689	11	0.000005

Levene's Test for Homogeneity of Variances (Poison
Effect: POISON*TREATMEN
Degrees of freedom for all F's: 11, 36

	MS Effect	MS Error	F	p
SURVIVAL	0.024601	0.005069	4.853537	0.000144

A reziduumok vizsgálata

Residuals1 fülön: Pred. & resid.



Box-Cox transzformáció

$$\sigma_y \sim \bar{y}^{\alpha} \qquad Var\left(y^{tr}\right)=\left(\frac{dy^{tr}}{dy}\right)^2 \sigma_y^2 = \left(\frac{dy^{tr}}{dy}\right)^2 y^{2\alpha}$$

$$Var\left(y^{tr}\right)= \text{konst} \quad \text{ha} \quad \frac{dy^{tr}}{dy} y^{\alpha} = \text{konst}$$

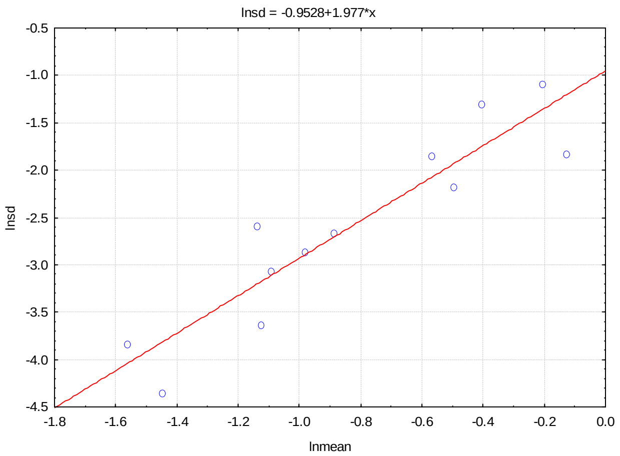
$$dy^{tr} = k y^{-\alpha} dy$$

$$y^{tr} = \int y^{-\alpha} dy = \begin{cases} y^{1-\alpha} & \text{ha } \alpha \neq 1 \\ \ln y & \text{ha } \alpha = 1 \end{cases}$$

$$y^{tr} = \int y^{-\alpha} dy = \begin{cases} y^{1-\alpha} & \text{ha } \alpha \neq 1 \\ \ln y & \text{ha } \alpha = 1 \end{cases}$$

α	λ=1-α	transzformáció
2	-1	$1/y$
1.5	-0.5	$1/\sqrt{y}$
1	0	$\ln y$
0.5	0.5	$\sqrt{y}$
0	1	(nincs transzformáció)

$\sigma_y \sim \bar{y}^{\alpha}$ $\qquad$ $\ln \sigma_y = k + \alpha \ln \bar{y}$ $\qquad$ egyenest kell illeszteni



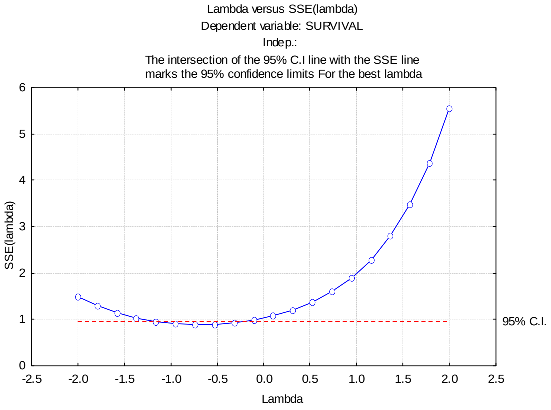
$\alpha \approx 2$

$\sigma_y \sim \bar{y}^{2}$

Box-Cox transzformáció

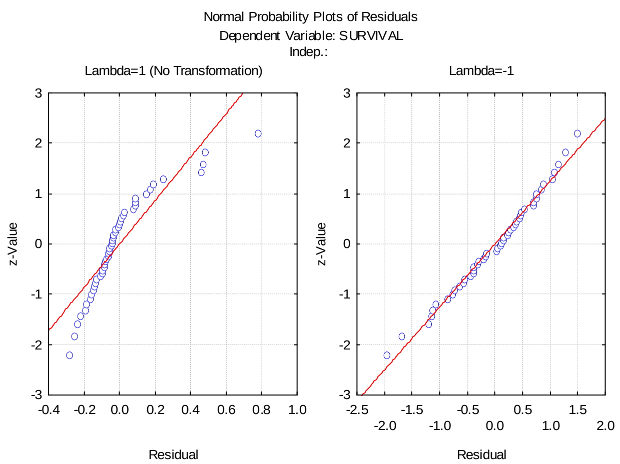
$$y^{tr} = y^{\lambda}$$

File>Open: (Program Files>StatSoft>Statistica8>Examples>Macros>
>Analysis Examples>BoxCox)

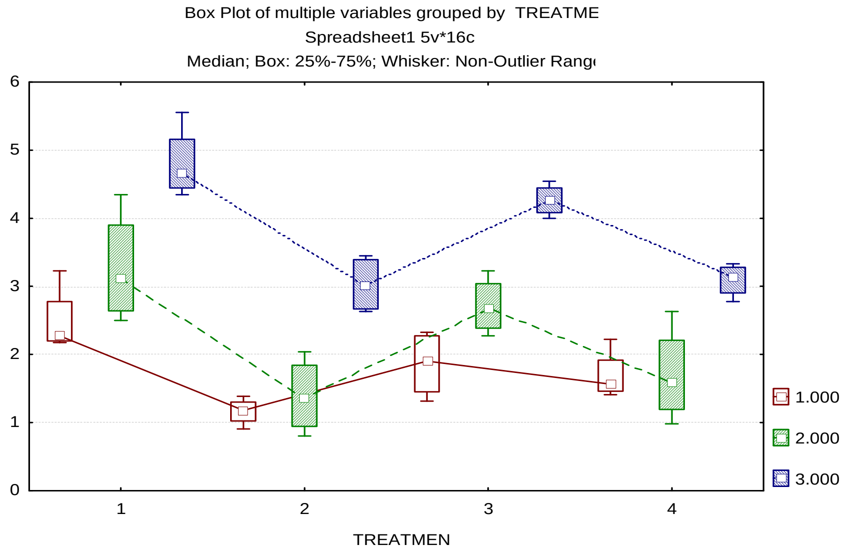


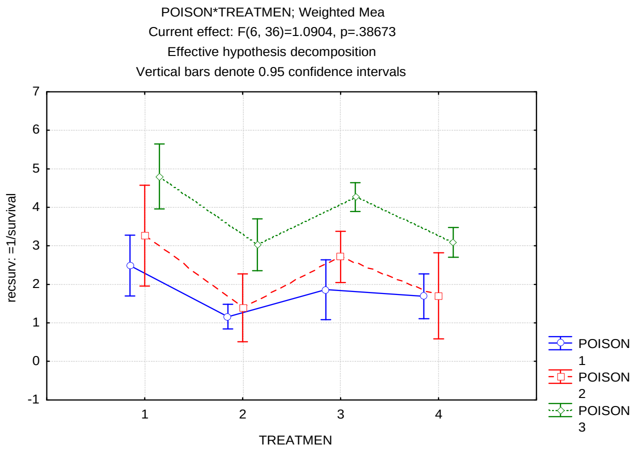
$$\lambda \approx -1$$

ANOVA 45



ANOVA 46


Box Plot of multiple variables grouped by TREATME
Spreadsheet1 5v*16c
Median; Box: 25%-75%; Whisker: Non-Outlier Rang
TREATMEN
1.000
2.000
3.000


POISON*TREATMEN; Weighted Mea
Current effect: F(6, 36)=1.0904, p=.38673
Effective hypothesis decomposition
Vertical bars denote 0.95 confidence intervals
recsurv: =1/survival
TREATMEN
POISON 1
POISON 2
POISON 3

Effect	Univariate Tests of Significance for Recsurv (Poison) Sigma-restricted parameterization Effective hypothesis decomposition SS	Degr. of Freedom	MS	F	p
Intercept	330.0892	1	330.0892	1374.881	0.000000
POISON	34.8771	2	17.4386	72.635	0.000000
TREATMEN	20.4143	3	6.8048	28.343	0.000000
POISON*TREATMEN	1.5708	6	0.2618	1.090	0.386733
Error	8.6431	36	0.2401		

A hatások még kifejezettebbek (*F* értékei nagyobbak), a kölcsönhatáshoz tartozó *p* 0.112 helyett 0.387 lesz.

A reziduumok vizsgálata